A Super Absorbing Boundary Condition for the Analysis of Waveguide Discontinuities with the Finite-Difference Method

Jian Yi Zhou and Wei Hong, *Member, IEEE*

***Abstract*—A super absorbing boundary condition is presented for the analysis of waveguide discontinuities with the finite-difference method (FDM). The discontinuity region is enclosed by the waveguide wall and two truncated planes at input and output waveguides, respectively. On the wall, homogeneous conditions are available. At mesh nodes in the region, finite-difference (FD) equations can be applied. When the truncated planes are far away from the discontinuity, the conventional dominant-mode transmission condition can be used for deducing the FD-type local equation at the terminated mesh nodes. A super absorbing boundary condition presented in this letter can be used to terminate the meshes very close to the discontinuity. It results in dramatic savings in computing time and memory needs.**

I. Introduction

The finite-difference method (FDM) is a simple and effective numerical method for the analysis of many kinds of discontinuities. The finite region containing the discontinuity is discretized by a finite-difference (FD) mesh, and the absorbing boundary condition (ABC) must be applied to the truncated mesh boundaries. If we use conventional dominant-mode transmission condition, the truncated mesh boundaries must be far away from the discontinuity. In this letter, a super ABC (SABC) is presented which can be used to terminate the meshes very close to the discontinuity and then results in dramatic savings in computing time and memory needs. The FD-type SABC is first assumed, then the coefficients are determined by measuring the SABC with dominant and several high-order modes, thus the SABC not only absorbs the dominant mode but also absorbs several high-order modes. It is something like the measured equation of invariance (MEI) [1], [2], but here we use the waveguide modes directly to measure the SABC, which avoids the integration of Green's function and the "metrons."

In this letter, the scattering characteristics of a rectangular waveguide loaded with a rectangular dielectric post are calculated by FDM and the SABC. Numerical results verify the validity of this technique.

II. Formulation

In this letter, the characteristic of a rectangular post with full height in a rectangular waveguide is analyzed. Assuming a TE_{10} wave of unit amplitude is incident upon the discontinuity, then only TE_{m0} modes are excited. For convenience, let $\varphi = E_y$, then φ satisfies the following Helmholtz equation:

$$\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \varphi}{\partial x^2} + k^2 \varphi = 0 \tag{1}$$

where $k^2 = \omega^2 \mu \varepsilon$.

Manuscript received December 9, 1996. This work was supported by NSF69625102.

The authors are with State Key Millimeter Wave Lab., Southeast University, Nanjing 210096, China.

Publisher Item Identifier S 1051-8207(97)03860-9.

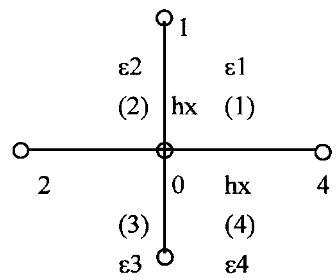


Fig. 1. Nodes for the FD equation.

The region enclosed by the waveguide wall and two truncated planes at input and output ports is discretized into a rectangular mesh. For interior nodes, a five-node FD equation (see Fig. 1) can be deduced as

$$\frac{h_z}{h_x}\varphi_1 + \frac{h_x}{h_z}\varphi_2 + \frac{h_z}{h_x}\varphi_3 + \frac{h_x}{h_z}\varphi_4 - 2\left(\frac{h_z}{h_x} + \frac{h_x}{h_z}\right)\varphi_0 + \frac{1}{4}h_x h_z\left(\sum_{i=1}^{4} k_i^2\right)\varphi_0 = 0. \tag{2}$$

In Fig. 1, the permittivity in each of the four quadrants can be different from each other, so the FD equation (2) is suitable for inhomogeneous dielectric.

III. SABC

As mentioned above, the FD equation (2) is adaptable for every interior node. At the waveguide walls, homogeneous boundary conditions are available. In order to terminate the meshes very close to the discontinuity and still strictly preserve the sparsity of FD equations, FD-type local SABC's are introduced. If the truncated planes are far enough from the discontinuity, the amplitudes of high-order modes excited by the discontinuity are negligible. Then, the conventional dominant-mode transmission condition is accurate enough, which exactly absorbs the dominant-mode scattered fields. When the truncated planes are very close to the discontinuity,

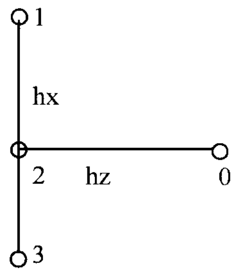


Fig. 2. Nodes for the SABC.

the conventional dominant-mode transmission condition fails. A local FD-type condition at terminated mesh nodes that can absorb both the dominant mode and several high-order modes is necessary.

Assume a FD-type local ABC at the truncated mesh boundaries may be represented by a local linear equation of the type

$$\varphi_0^s + c_1\varphi_1^s + c_2\varphi_2^s + c_3\varphi_3^s = 0 \tag{3}$$

where the nodes configuration is shown in Fig. 2 and the coefficients $c_i(i = 1, 2, 3)$ are to be determined.

In order to find the coefficients, we use the scattered fields of the dominant mode and the first two high-order modes to measure the SABC (3). Then, the following matrix equation with respect to the unknown coefficients are obtained:

$$\begin{bmatrix} \varphi_1^{(1)} & \varphi_2^{(1)} & \varphi_3^{(1)} \\ \varphi_1^{(2)} & \varphi_2^{(2)} & \varphi_3^{(2)} \\ \varphi_1^{(3)} & \varphi_2^{(3)} & \varphi_3^{(3)} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -\varphi_2^{(1)} e^{-jk_1 h_z} \\ -\varphi_2^{(2)} e^{-jk_2 h_z} \\ -\varphi_2^{(3)} e^{-jk_3 h_z} \end{bmatrix}. \tag{4}$$

Then, the coefficients $c_i(i = 1, 2, 3)$ are determined from its solutions.

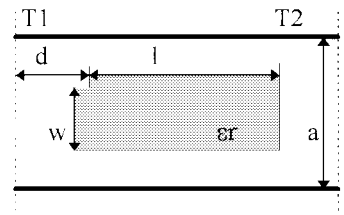
IV. Results and Discussion

A. Results of Some Discontinuities

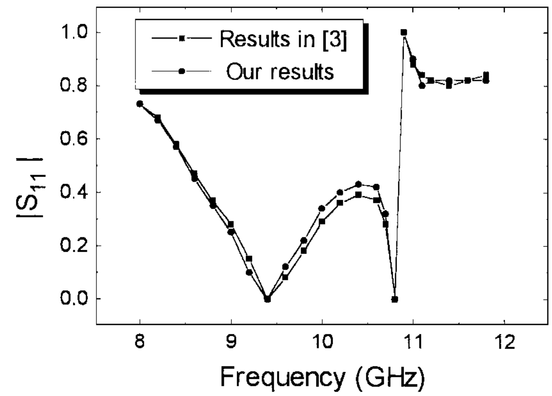
Fig. 3(b) shows the reflection coefficients of a waveguide loaded with a rectangular dielectric post whose configuration is shown in Fig. 3(a). The results agree well with those in [3].

B. Comparison Between the ABC's

Fig. 4 shows the numerical results calculated with the SABC, the conventional ABC (CABC), and the modified second-order Mur's ABC. The configuration is shown in Fig. 3(a), where $a = 7.112$ mm, $w = 4$ mm, $l = 2$ mm, and $\varepsilon_r = 8.2$. The calculation was carried out around the resonant frequency (about 33 GHz), where the high-order modes were excited strongly. Because of the symmetry, the TE_{10}, TE_{30}, and TE_{50} modes were used to determine the coefficients of the SABC. The high-order modes attenuate along the waveguide. The higher the mode is, the faster it attenuates. When the distance between the discontinuity and the ABC (denoted as d) was large, the high-order modes attenuated adequately and their effects were negligible; all of these ABC's were accurate. When d became smaller, the effects of the first two high-order modes were no longer negligible, and the SABC successfully obtained accurate results whereas the CABC and the Mur's ABC failed because they cannot absorb the high-order modes adequately. It can be found from Fig. 4 that the



(a)



(b)

Fig. 3. Configuration and reflection of centrally put rectangular post. $A = 22.86$ mm, $w/a = 0.524$, $l/a = 0.262$, and $\varepsilon_r = 8.2$.

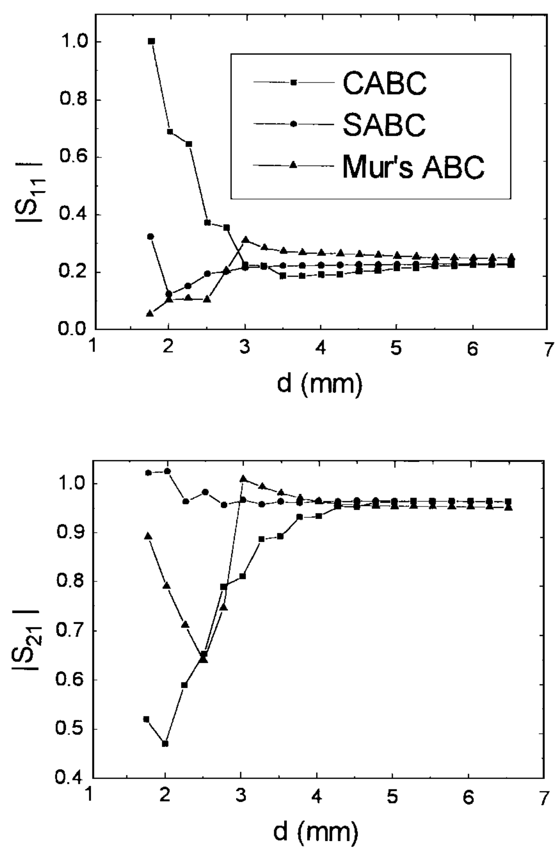


Fig. 4. S-parameters versus d calculated with different ABC's.

SABC keeps its accuracy until d is very small that higher order modes (higher than TE_{50} mode) are so strong that they should be taken into account. We can conclude that the SABC can be set much closer to the discontinuity than the CABC and the Mur's ABC, without loosing any accuracy. Great computational memory and time were saved when the SABC was applied.

V. Conclusion

The SABC we developed for the analysis of the waveguide discontinuity with FDM is proved to be valid and efficient. The coefficients of the assumed FD-type local SABC are determined by the measurement with the dominant mode and the first two high-order modes. The determination is so simple that very little additional computing time is needed. Numerical results have shown the superiority of the SABC to several other ABC's.

References

[1] K. K. Mei *et al.*, "The measured equation of invariance: A new concept in field computation," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 320–327, Mar. 1994.

[2] W. Hong, Y. W. Liu, and K. K. Mei, "Application of the measured equation of invariance to solve scattering problems involving penetrable medium," *Radio Sci.*, vol. 29, no. 4, 1994.

[3] J. Abduluour and L. Marchildon, "Scattering by a dielectric obstacle in a rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1988–1994, 1993.

[4] G. Mur, "Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic field equations," *IEEE Trans. Electromagn. Compat.*, vol. EC-23, pp. 377–382, Nov. 1981.